SAFETY LEAD MATERIAL SELECTION

M. Kuchnir

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INTRODUCTION

The Energy Doubler superconducting magnets in their present design require safety leads for prompt extraction of the stored energy in the event of a quench. The design criteria for these leads differ from the one for the power leads since the safety lead will carry current only during infrequent and short periods.

In this memo, the problem is analyzed, a key parameter involved (the integral of the specific heat over the resistivity from room temperature to a still safe temperature T_{max}) is calculated for several materials using solid state theory and Debye Temperature values from the literature. This parameter is useful when comparing the capacity of metals for handling current surges. The computer programs developed to do these calculations are included as an appendix.

ANALYSIS

A conservative design criterion is that the energy deposited in any section of the lead be insufficient to raise its temperature above the safe value T_{max} under the worst external cooling conditions; namely, no external cooling.

Let A, ρ, and c be the cross section area, resistivity and specific heat of an element of length dx of the lead. The electrical resistance of this element will be $R = \rho(T)\frac{dx}{A}$ and its heat capacity $C = \mu A c(T) dx$ where μ is the density of the material. When a time dependent current $I = I(t)$ goes through the lead, the energy deposited in the lead per unit time by Joule heating raises the temperature, T, of the lead element according to the energy balance expression:

$$A\mu \; c(T) \frac{dT}{dt} dx = \left(\frac{1}{A}\right) I^2(t)\rho(T)dx. \tag{1}$$

According to the above mentioned criterion we are neglecting heat transfer into or away from this element of lead. The usual short duration, Δt, of $I(t) \neq 0$ validates this approximation which in any case represents the worse case from a safety point of view.

Further simplification of the problem is made by assuming a uniform cross section lead, which is the preferred type from the standpoint of fabrication.

Expression (1) in an integral form reads:

$$A^2\mu \int_{T_{initial}}^{T_{max}} \frac{c(T)}{\rho(T)} dT = \int_0^{\Delta t} I^2(t) \; dt. \tag{2}$$

The right hand side could be called a "quench load", and is expressed in A^2sec or Joules/Ohms. In fuse literature it is known by the misleading name of fuseing current. A maximum I(t) expected in the quench of an Energy Doubler dipole is graphically shown in Fig. 1. Its corresponding quench load is:

$$F = \int_0^{\Delta t} I^2(t)dt = 6.66 \times 10^6 \text{ Joule}/\Omega.$$

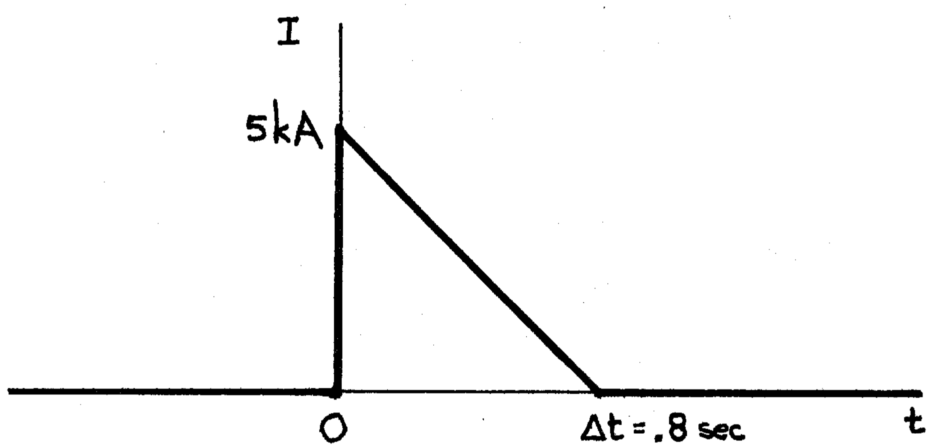

Figure 1

Maximum Current Pulse Expected From a Quench of an Energy Doubler Dipole

QUENCH CAPABILITY

The left hand side of expression (2) depends only on the lead material, its cross section and the temperature limits. By specifying the initial temperature as room temperature (the highest temperature before the current surge) and T_{max} as the highest safe temperature, we define the "quench capability", f, of the lead material as:

$$f = \mu \int_{300K}^{T_{max}} \frac{c(T)}{\rho(T)}\, dT.$$

According to solid state theory both the specific heat and the resistivity as a function of temperature can be expressed in terms of the Debye Temperature, Θ, of the material and its resistivity at this temperature, $\rho(\Theta)$.

The specific heat of a solid near and above room temperature is mainly due to lattice vibrations and according to the Debye theory[1] is given by

$$c(T) = \frac{9k_B N}{M}\left(\frac{T}{\Theta}\right)^3 \int_0^{\Theta/T} \frac{e^y y^4 dy}{(e^y-1)^2}$$

where k_B = Boltzmann constant,

N = Avogadro's number,

and M = Molecular weight of material.

Also the resistivity can be expressed in terms of a dimensionless integral[2] when the electrons are scattered mainly by phonons:

$$\rho(T) = \rho(\Theta)\left(\frac{T}{\Theta}\right)^5 \int_0^{\Theta/T} \frac{y\,dy}{(e^y-1)(1-e^{-y})}$$

One of the computer programs in the appendix, SL3.F4, calculates the above integrals and their ratio to within .2%, generating a table SL4.DAT which is used in the second program SL4.F4. This second program requests data on the material (μ, M, Θ, $\rho(\Theta)$ and the thermal conductivity integral $\int_{4.2K}^{300K} \lambda dT$) and calculates the quench capability, f, as well as the merit figure z explained below.

When a safety lead is <u>not</u> cooled by exchanging heat with a gas flow, the conduction heat load, Q, should be minimized.

From $Q = \frac{A}{\ell}\int_{4.2K}^{300K} \lambda dT$ where λ is the thermal conductivity and

ℓ is the length of the lead, and $F = A^2 . f$ we get

$$Q = \sqrt{\frac{F}{f}} \, \frac{1}{\ell} \int_{4.2K}^{300K} \lambda dT \, .$$

For a fixed design, the material with largest merit figure:

$$z = \sqrt{f} \Big/ \int_{4.2K}^{300K} \lambda dT$$

should be preferred if all other mechanical and economic constraints are satisfied.

However, vapor cooled leads are very efficient in reducing the conduction heat load and a material with largest f compatible with other fabrication constraints should be selected.

On Table I, the parameters f and z are presented for a list of single element materials in the case of a heating excursion from room temperature to 180°C (soft solder lowest melting point is 185°C). The material properties used as input were obtained from the literature[3], which in many cases does not have unique values for Θ or the thermal conductivity integral. Average values were then used and the percental error indicated in f covers the spread found in the literature.

On Fig. 2, the quench capability of copper and nickel are shown as a function of the upper limit of the heating excursion. The third program of the Appendix, SL5.F4, calculates this result.

CONCLUSION

The materials examined classify themselves with respect to quench capability inversely to their resistivity at the Debye Temperature, and copper seems an easy choice when the conduction heating is compensated by vapor cooling. Lead is an exception.

For leads with uniform cross section and without vapor cooling niobium followed by nickel are better choices since lead might be too soft.

Copper plated stainless steel, might be the best engineering compromise for simple vapor cooled leads.

ACKNOWLEDGEMENT

The interest and cooperation of R. H. Flora in this work was sincerely appreciated.

REFERENCES

[1] See for instance "Physics of Electronic Conduction in Solids" by Frank J. Blatt, p. 46, McGraw Hill Book Company (1968).

[2] Ibid, p. 189.

[3] Ibid, p. 48 and 192 for Θ; Handbook of Chemistry and Physics for μ and M; L. A. Hall, NBS Technical Note 365 for $\rho(\Theta)$; R. B. Stewart and V. J. Johnson, WADD Technical Report 60-56 for $\int_{4.2K}^{300K} \lambda dT$

TABLE I

Quench Capability and Merit Figure of Several Elements

For A Heating Excursion From 300 To 453K

Material	f $MJ/\Omega cm^4$	z $sec^{1/2}\ cm^{-1}V^{-1}$
Au	630.±5%	.022
Ag	557.±2%	.010
Cu	319.±5%	.012
Pb	143.±14%	.075
Al	96.2±1%	.0063
W	89.5±7%	.004
Mo	48.6±9%	.014
Ta	46.6±8%	.038
Ni	44.6	.031
Nb	41.5±5%	.041
Fe	31.7±5%	.014

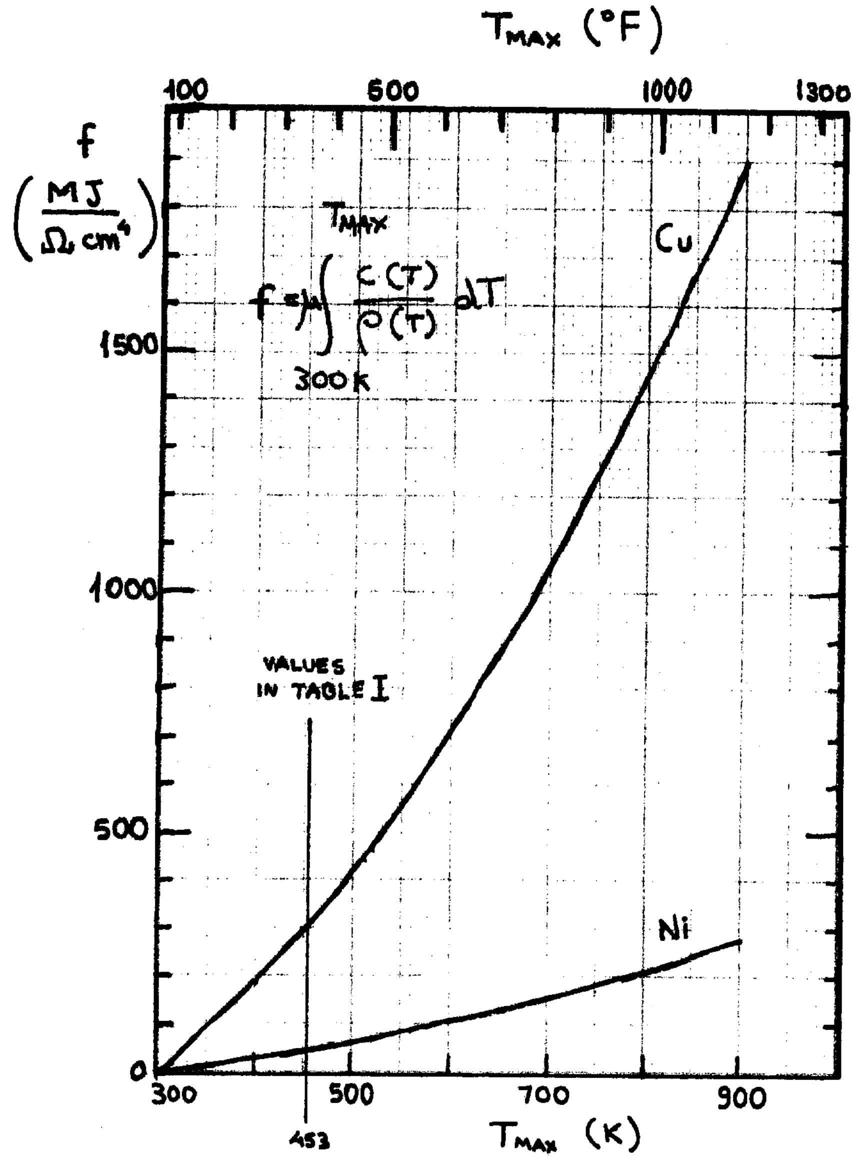

Figure 2

Quench Capability of Copper and Nickel

APPENDIX

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C          SAFETY LEAD MATERIAL SELECTION SCHEME
C
C SL3:     BASED IN THE DEBYE THEORY THIS PROGRAM GENERATES TABLES OF
C          SPECIFIC HEAT, RESISTIVITY AND THEIR RATIO IN A DIMENSIONLESS
C          FORM. ITS OUTPUT, SL4.DAT, IS TO BE USED BY SL4.F4.
C          EXECUTION REQUEST:  EXEC SL3,DSKB:QATR[10,6]
C          PRECISION OF RATIO: .2%
C
           DIMENSION C(2500),R(2500),CR(2500),AUX(100)
           EXTERNAL SH,RHO
           XL=0.
           EPS=5.E-6
           EPSR=1.E-6
           NDIC=5
           NDIR=5
           IER=3
           Y=0.
           IA=0
           CALL OFILE(20,'SL4.DAT')
           WRITE(20,6)
C GENERATE TABLES OF INTERNAL INTEGRALS AND RATIO
           DO 1 I=150,2200,1
C          REPORT CALCULATION PROGRESS TO TERMINAL
           AI=I
           II=AI/10.
           IF(II.NE.IA)WRITE(5,7)I
           IA=II
7          FORMAT('  ',I4)
           XI=I
           XU=XI/1000.
           J=I-149
31         CALL QATR(XL,XU,EPSC,NDIC,SH,Y,IER,AUX)
           EPSC=1.E-3*Y
           C(J)=Y
           IF(IER.EQ.2) GO TO 30
           IF(IER.NE.0)WRITE(5,2)IER,XU,Y,NDIC
2          FORMAT('  QATR ERROR',I2,'  SH(',F6.3,')=',F,I)
33         CALL QATR(XL,XU,EPSR,NDIR,RHO,Y,IER,AUX)
           EPSR=1.E-3*Y
           R(J)=Y
           IF (IER.EQ.2) GO TO 32
           IF(IER.NE.0)WRITE(5,3)IER,XU,Y,NDIR
3          FORMAT('  QATR ERROR',I2,'  RHO(',F6.3,')=',F,I)
           CR(J)=C(J)/R(J)
           WRITE(20,5)XU,C(J),R(J),CR(J)
1          CONTINUE
C PRINT  SUMARY TABLE
           WRITE(5,6)
6          FORMAT('          TD/T       C/ATOM  RESISTIVITY     C/R')
           DO 4  I= 200,1500,100
```

```
           XI=I
           X=XI/1000.
           J=I-149
           WRITE(5,5) X,C(J),R(J),CR(J)
     5     FORMAT('   ',F11.4,2E11.3,F11.4)
     4     CONTINUE
           STOP
     C INCREASE QATR PRECISION
     30    IF(NDIC.GE.100) GOTO 31
           NDIC=NDIC+1
           WRITE(5,34)NDIC,XU
     34    FORMAT('  NDIC INCREASED TO',I3,' AT X=',F)
           GO TO 31
     32    IF(NDIR.GE.100) GOTO 33
           NDIR=NDIR+1
           WRITE(5,35)NDIR,XU
     35    FORMAT('  NDIR INCREASED TO',I3,' AT X=',F)
           GOTO 33
           END
     C DEBYE THEORY FUNCTIONS
           FUNCTION SH(X)
           SH=0.
           IF(X.EQ.0.)RETURN
           Y=EXP(X)
           SH=Y*X**4/((Y-1.)**2)
           RETURN
           END
           FUNCTION RHO(X)
           SH=0.
           IF(X.EQ.0.)RETURN
           RHO=X**5/((EXP(X)-1.)*(1.-EXP(-X)))
           RETURN
           END
```

PRINTOUT OF SUMMARY TABLE

2190
2200

TD/T	C/ATOM	RESISTIVITY	C/R
0.2000	0.266E-02	0.399E-03	6.6642
0.3000	0.896E-02	0.202E-02	4.4440
0.4000	0.212E-01	0.635E-02	3.3343
0.5000	0.412E-01	0.154E-01	2.6688
0.6000	0.707E-01	0.318E-01	2.2254
0.7000	0.112E+00	0.585E-01	1.9088
0.8000	0.165E+00	0.989E-01	1.6716
0.9000	0.233E+00	0.157E+00	1.4873
1.0000	0.317E+00	0.237E+00	1.3400
1.1000	0.418E+00	0.343E+00	1.2196
1.2000	0.537E+00	0.479E+00	1.1194
1.3000	0.674E+00	0.651E+00	1.0347
1.4000	0.831E+00	0.864E+00	0.9622
1.5000	0.101E+01	0.112E+01	0.8995

(Table SL4.DAT available on request)

```
END OF EXECUTION
CPU TIME: 56:22.95      ELAPSED TIME: 2:31:46.83
EXIT
```

```
C        SAFETY LEAD MATERIAL SELECTION SCHEME
C
C SL4:   THIS PROGRAM USES THE OUTPUT TABLE SL4.DAT
C        GENERATED BY SL3.F4, REQUESTS MATERIAL DATA AND TEMPERATURE
C        LIMITS AND CALCULATES THE MATERIAL DEPENDENT
C        INTEGRAL F AND THE MERIT INDEX Z
C
         DIMENSION C(2500),R(2500),CR(2500)
         CALL IFILE(1,'SL4.DAT')
         READ(1,6)
6        FORMAT('
         DO 1 I=150,2200,1
         J=I-149
         READ(1,5)XU,C(J),R(J),CR(J)
5        FORMAT('  ',F11.4,2E11.3,F11.4)
1        CONTINUE
9        WRITE(5,7)
7        FORMAT('  ENTER  TI AND TM (INITIAL AND MAXIMUM TEMP (K)')
         READ(5,8)TI,TM
8        FORMAT(2G)
         FLAG=0.
         IF(TI.EQ.0)STOP
15       IF(FLAG.EQ.1.)GOTO 3
         WRITE(5,11)
11       FORMAT('  ENTER: ATOMIC WEIGHT(G), DENSITY(G/CM3), TDEBYE(K),'//
         1          RESISTIVITY(E-6.OHM.CM) AT TDEBYE,'//          THERMAL
         2CONDUCTIVITY INTEGRAL(WATT/CM) FROM 4 TO 300K.'//
         3  (FOLLOWING PROMPTERS WILL JUST ASK FOR:A,D,TD,RTD,TCI)')
         FLAG=1.
16       READ(5,12) A,D,TD,RTD,TCI
12       FORMAT(5G)
13       IF (A.EQ.0) GOTO 9
         GG=17.706*D*TD/(A*RTD)
         XL=TD/TM
         XU=TD/TI
         WRITE(5,2)XL,XU
2        FORMAT('  XL=',F11.4,'   XU=',F11.
         IF(XL.LT..15.OR.XU.GT.2.2) GOTO 9
         LL=XL*1000.-149
         LU=XU*1000.-148
         H=.001*.5
         K=LL
C INTEGRATION LOOP
         SUM=0.
10       K=K+1
         X22=K+149
         X22=(1000./X22)**2
         X21=K+148
         X21=(1000./X21)**2
         SUM1=SUM
         SUM=SUM+H*(CR(K)*X22+CR(K-1)*X21)
         IF(K.LT.LU) GOTO 10
         F=GG*SUM
         Z=SQRT(F)/TCI
         WRITE(5,14) F,Z
14       FORMAT('  QUENCH CAPABILITY=',F6.2,'JOULES/(OHM*CM4)    MERIT
         1 VALUE=',F6.4)
         GOTO 15
3        WRITE(5,4)
4        FORMAT('  ENTER:A,D,TD,RTD,TCI')
         GO TO 16
         END
```

```
C         SAFETY LEAD MATERIAL SELECTION SCHEME
C
C SL5:    THIS PROGRAM USES THE OUTPUT TABLE SL4.DAT GENERATED BY SL3.F4
C         FOR TI=300.K AND GENERATES F(TM).
C         MATERIAL PARAMETERS ARE REQUESTED LIKE IN SL4 FAST PROMPTER.
C         ZERO ENTRY TERMINATES RUN.
C
          DIMENSION C(2500),R(2500),CR(2500)
          CALL IFILE(1,'SL4.DAT')
          READ(1,6)
6         FORMAT('
          DO 1 I=150,2200,1
          J=I-149
          READ(1,5)XU,C(J),R(J),CR(J)
5         FORMAT('  ',F11.4,2E11.3,F11.4)
1         CONTINUE
          A=63.5
          D=8.96
          TD=343.
          RTD=1.85
          TCI=1500.
3         WRITE(5,4)
4         FORMAT('  ENTER:A,D,TD,RTD,TCI')
          READ(5,12)A,D,TD,RTD,TCI
12        FORMAT(5G)
          IF (A.EQ.0.) STOP
          TI=300.
          WRITE(5,7)
7         FORMAT('  TI=300.K'//'    TM  (K)     F  (MJ/OHM.CM4)   Z')
          DO 30 KK=310,900,10
          TM=KK
          GG=17.706*D*TD/(A*RTD)
          XL=TD/TM
          XU=TD/TI
          IF(XL.LT..15.OR.XU.GT.2.2) GOTO 9
          LL=XL*1000.-149
          LU=XU*1000.-148
          H=.001*.5
          K=LL
C INTEGRATION LOOP
          SUM=0.
10        K=K+1
          X22=K+149
          X22=(1000./X22)**2
          X21=K+148
          X21=(1000./X21)**2
          SUM1=SUM
          SUM=SUM+H*(CR(K)*X22+CR(K-1)*X21)
          IF(K.LT.LU) GOTO 10
          F=GG*SUM
          Z=SQRT(F)/TCI
          WRITE(5,14) TM,F,Z
14        FORMAT('  ',F5.0,F12.2,F13.4)
30        CONTINUE
          GOTO  3
9         WRITE(5,2)XL,XU
2         FORMAT('  XL=',F11.4,'     XU=',F11.4)
          STOP
          END
```